Exam E (Part II) **Name**

Please Note: Present you solutions in a well organized, legible way. Calculators may be used *only* in elementary computational mode, and not in calculus mode (even for exploratory purposes).

1a. Let $f(x)$ be a function that is continuous for all x and let $F(x)$ be any anti-derivative of $f(x)$. Explain why $F(x+dx) - F(x) \approx f(x) \cdot dx$ for any very small number dx.

1b. Check the approximation in the situation $f(x) = x^2$, $F(x) = \frac{1}{3}x^3$, $x = 2$, and $dx = 0.001$.

$f(x)\,dx \approx$	$F(x+dx) - F(x) \approx$

2. Consider the function $f(x) = x^2(x-7)^{\frac{1}{3}}$.

i. Verify (show all steps) that $f'(x) = \frac{7(x^2-6x)}{3(x-7)^{\frac{2}{3}}}$.

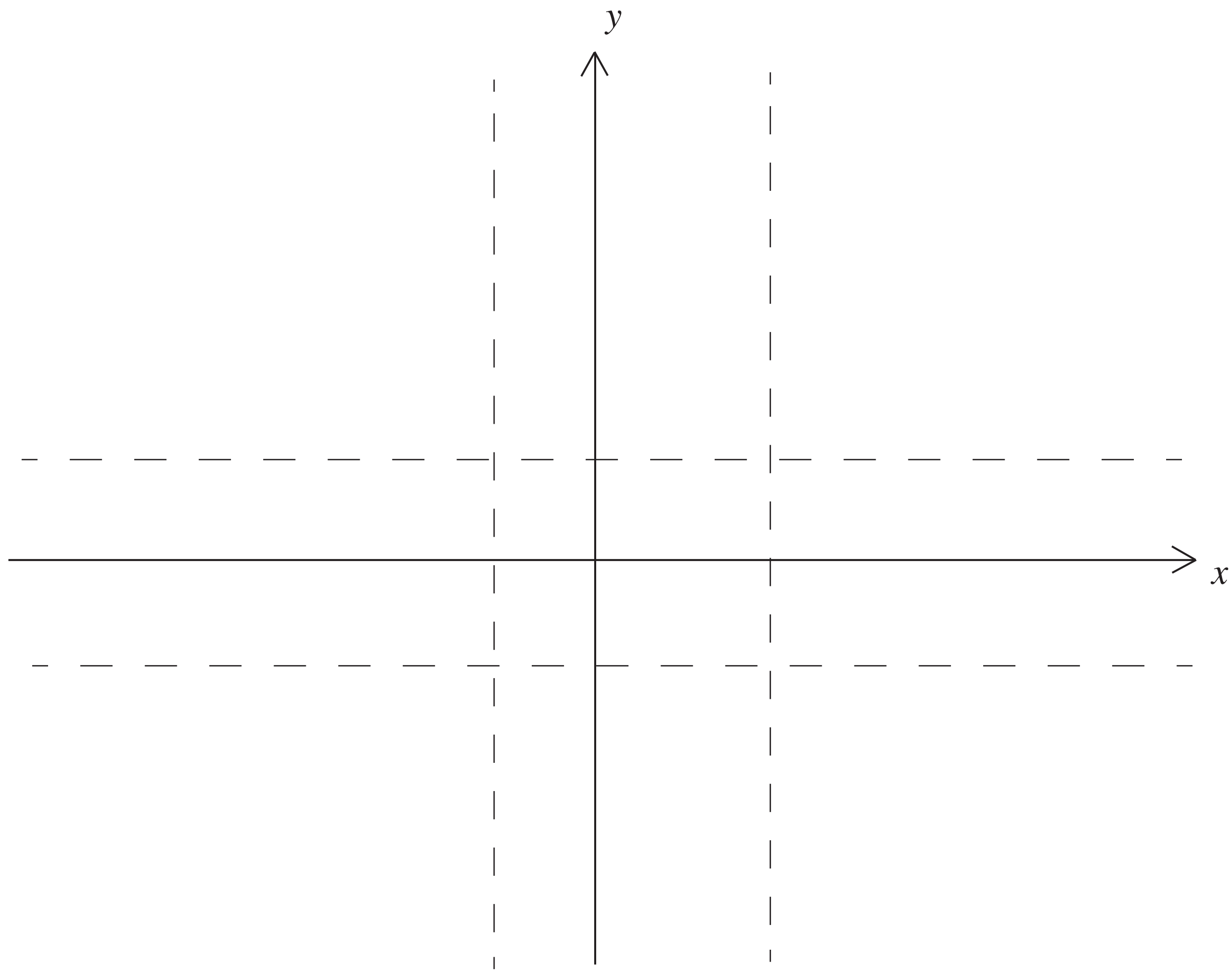
ii. Locate the critical numbers for $y = f(x)$ on the number line below.

x

At what points on the graph does $y = f(x)$ have horizontal or vertical tangents?

iii. Determine the interval(s) over which f is increasing and those over which f is decreasing, as well as the local maxima and minima of f.

3. You are given a function that has the lines $y = 1$ and $y = -1$ as horizontal asymptotes and the lines $x = -1$ and $x = 2$ as vertical asymptotes. The function also satisfies $f'(x) > 0$ for $x < -1$



and $x > 2$, and $f'(x) < 0$ for $-1 < x < 2$, as well as $f''(x) > 0$ for $x < -1$ and $-1 < x < 1$ and $f''(x) < 0$ for $1 < x < 2$ and $2 < x$. In the space provided below draw a careful sketch of a graph that could be the graph of $y = f(x)$.

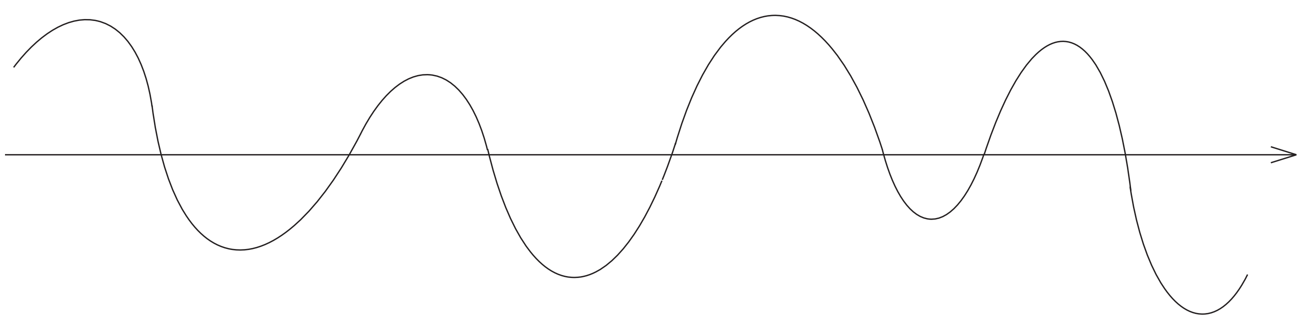
4. Consider the long sum

$$(2)^3 \cdot \frac{1}{1000} + (2 + \frac{1}{1000})^3 \cdot \frac{1}{1000} + (2 + \frac{2}{1000})^3 \cdot \frac{1}{1000} + (2 + \frac{3}{1000})^3 \cdot \frac{1}{1000} + \cdots + (4 + \frac{999}{1000})^3 \cdot \frac{1}{1000}\,.$$

This sum can be approximated by the definite integral of a function. What is the function, what is the integral, and what is the approximate value of the sum?

$f(x) =$	sum $\approx$

5. The graph of a typical continuous function $f(x)$ that is defined for all x is shown below. Select some number c on the x-axis and use it to define an area function $A(x)$ for $f(x)$.



a. Select some number c on the x-axis and use it to define an area function $A(x)$ for $f(x)$.

$A(x) = \Bigg\{$

b. What is the important property that the function $A(x)$ has (in the context of calculus)?

c. Take the special case $f(x) = x^3$ and $c = 2$ and determine the function $A(x)$ explicitly.

$A(x) =$

6. Evaluate $\int x \tan^{-1} x^2 \, dx$. (Suggestion: use both integration by substitution and by parts.)

7. Find a solution $y = f(x)$ of the equation $y' + 3x^2 y = 6x^2$ that satisfies $f(0) = 4$.

$y =$

8. A falling ball has two forces acting on it both in the vertical direction. The gravitational force is given by $W = mg$, where m is the mass of the ball and g is the gravitational constant. The drag is $D = cv^2$, where v is the velocity and c a constant of proportionality. Use Newton's second law to set up a differential equation that the motion of the ball satisfies.